

second



BIRZEIT UNIVERSITY

Mathematics Department

STAT236 – Second-Hour Exam - Fall 2011

33  
40

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Section:

1	Mohammad Madiah	MW 14:00 – 15:20
2	Tareq Sadeq	SMW 10:00 – 10:50
3	Tareq Sadeq	SMW 13:00 – 13:50
4	Hani Kabajah	SMW 12:00 – 12:50
5	Hani Kabajah	SMW 09:00 – 09:50
6	Maher Abdellatif	TR 09:30 – 10:50
7	Hani Kabajah	SMW 08:00 – 08:50

Formulas:

- Binomial:  $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$ ,  $E(x) = np$ ,  $\sigma = \sqrt{np(1-p)}$

- Poisson:  $f(x) = \frac{\mu^x e^{-\mu}}{x!}$

- Exponential:  $F(x_0) = P(X \leq x_0) = 1 - e^{-\frac{x_0}{\mu}}$

- Standard error of the sample mean:  $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$  (Finite population)

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \text{ (Infinite population)}$$

- Standard error of the sample proportion:  $\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n} \cdot \frac{N-n}{N-1}}$  (Finite population)

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} \text{ (Infinite population)}$$

$$E = np$$

Question 1: Find the correct answers

If  $X$  is a discrete random variable with a probability function. Questions 1-2.

$$f(x) = \frac{x}{10}, \quad x=1,4,5 \quad \text{PDF}=1$$

1. Find  $E(X)$ .  
 (a) 4.2      b. 6      c. 3.8      d. 1.36

2. Find  $\text{Var}(X)$ .  
 (a) 4.2      b. 1.56      c. 0.89      (d) 1.36

A sample of 10 Palestinians is randomly and independently selected. If 30% of the Palestinian population are unemployed, Questions 3-5

3. What is the mean number of unemployed persons?

- a. 2      b. 8      c. 1.6      (d) 3

4. What is the probability that 5 persons are unemployed?

- (a) 0.103      b. 0.0264      c. 0.107      d. 0.5

5. What is the probability that at least 2 persons are unemployed?

- a. 0.121      (b) 0.879      c. 0.3754      d. 0.6246

6. An airport administration desires to study the waiting time in a line at check-in desk. The time follows an exponential distribution with a mean of 10 minutes. The probability that the waiting time is less than 15 minutes is:

- a. 0.2231      b. 0.7135      (c) 0.2865      (d) 0.7769

A manufacturer produces keys at the rate of 10 keys per hour. Questions 7-9

7. What is the probability that the manufacturer produces exactly 8 keys in an hour?

- a. 0.063      b. 0.0076      c. 0.0413      (d) 0.11

8. What is the probability that the manufacturer produces exactly 2 keys in 15 minutes?

- (a) 0.2565      b. 0.3      c. 0.0076      d. 0.2137

9. What is the probability that the time needed to produce a key is more than 10 minutes?

- (a) 0.1889      b. 0.2      (c) 0.1      d. 0.3679

10. A computer randomly selects numbers between 4 and 10 with uniform probability distribution. What is the probability that a number will have a value of at least 5?

- a. 0.25      b. 0.33      (c) 0.833      d. 0.625

$$\frac{1}{6}$$

$$x \geq 5$$

0, elsewhere.

$$4 < x \leq 10$$

$$\frac{10 - 4}{10 - 4} = \frac{6}{6} = 1$$

$$\frac{5 - 4}{10 - 4} = \frac{1}{6} = 0.1667$$

$$\frac{1}{6}$$

$\mu = 900$        $\sigma = 40$

The wages of employees follow a normal distribution with a mean of \$900 and a standard deviation of \$40. Questions 11 - 14

11. What is the probability that a selected employee will have a wage greater than \$870?  $+ > \frac{870 - 900}{40}$   
 a. 0.25      b. 0.8944      c. 0.1056      **d. 0.7734**       $1 - 0.2266 = 0.7734$

12. What is the probability that a selected employee will have a wage between \$1000 and \$1018?  
 a. 0.9876      b. 0.9938      c. 0.0062      **d. 0.0046**       $1 - (0.9876 + 0.9938) = 0.0162$   
 $\frac{0.0162}{2} = 0.0081$

13. What is the probability that a selected employee will have a wage in the top 5% of wages?  
 a. 0.9      **b. 0.05**      c. 0.1      d. 0.2

14. An employee has a wage among the top 5% of wages if her wage is higher than:  
**a. \$965.79**      b. \$900      c. \$940.5      d. \$951.26       $900 + (0.05 \times 40) = 920$

15. Palestine Central Bureau of Statistics selects a sample, where 20% of the sample individuals are unemployed. This is the same as the population unemployment rate. This sampling method is called:

- a. Simple-random sampling
- b. Systematic sampling
- c. Stratified sampling
- d. Convenience sampling
- e. Judgment sampling

16. A professor at BZU studies the determinants of students grades. The professor asks some students to select random samples of BZU students and let them fill a questionnaire. This sampling method is called:

- a. Simple random sampling
- b. Systematic sampling
- c. Stratified sampling
- d. Convenience sampling
- e. Judgment sampling

**Question 2:**

State whether each of the following is a **discrete** or **continuous** random variable?

- a. The number of customers arriving at a bank: discrete Random variable
- b. The wages of employees at a company: discrete Continuous Random variable
- c. The number of customers contacted per day by a company: discrete
- d. The time passed during an interviews: Continuous Random variable



Question 3:

Assume you know that 10% of STAT236 students fail the course. A class of 100 students is selected.

- a) What is the mean number of failures in the sample?

$$\mu = n \cdot p = 100 \cdot 0.1 = 10$$

- b) What is the standard deviation of failures?

$$\sigma = \sqrt{n \cdot p \cdot (1-p)} = \sqrt{100 \cdot 0.1 \cdot (1-0.1)} = 100 \cdot 0.1 \cdot 0.9 = \sqrt{9}$$

- c) Check if the conditions of normal approximation of binomial probabilities are satisfied.

$$n \cdot p = 100 \cdot 0.1 = 10 > 5$$
$$n \cdot (1-p) = 100 \cdot 0.9 = 90 > 5$$

Can use normal approx. of Binomial

- d) What is the approximate probability that at most 6 students will fail in the sample of 100 students?

$$\mu = n \cdot p = 10$$

$$\sigma = \sqrt{n \cdot p \cdot (1-p)} = 3$$

$$P(X \leq 6) = P\left(Z \leq \frac{6.5 - 10}{3}\right) = P(Z \leq -1.17) = 0.1210$$

Question 4:

8 ~~100~~ / 100

In 2006, the participation rate in Palestinian elections was 70%. A survey of 100 voters was conducted to estimate the proportion of those who participated at elections.

a) What is the probability that one randomly selected individual will have participated in elections?

$n \cdot p = 100 \times .70 = 70 \geq 5$  normal Dist.  
 $n(1-p) = 100 \times .30 = 30 \geq 5$

b) Find the expected value of the sample proportion.

$E(\bar{p}) = p = .70$

c) Find the standard error of the sample proportion.

Standard error =  $\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.70(.30)}{100}} = 0.0458$

d) What is the probability that the sample proportion will be within  $\pm 0.08$  of the population proportion?

$n \cdot p = 70$

$Var = n \cdot p \cdot (1-p) = 100 \cdot .70 \cdot .30 = 21$

$\sigma = \sqrt{Var} = \sqrt{21} = 4.5$

$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = .0458$

$P(-.08 \leq \bar{p} - p \leq .08) \Rightarrow P\left(\frac{-0.08}{0.0458} \leq Z \leq \frac{0.08}{0.0458}\right)$   
 $= P(-1.74 \leq Z \leq 1.74)$   
 ~~$= P(0.49 \leq Z \leq 1.74)$~~

$= P(Z \leq 1.74) - P(Z \leq -1.74)$

$= 0.9595 - 0.0409$